Middle School Math Camp - bridge2math.org

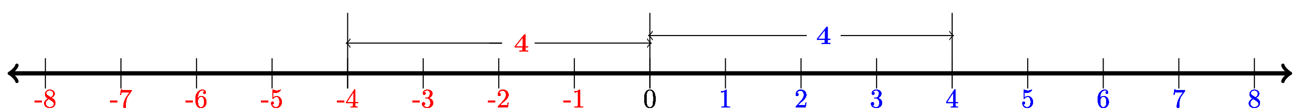
January 4, 2025

1 Absolute Values

The absolute value of a number is its distance from zero on the number line. It is always a non-negative value, regardless of whether the original number is positive or negative. For example, the absolute value of -5 is 5, and the absolute value of 5 is also 5.

The absolute value of a number is represented by vertical bars around a number. For example, $|x|$ represents the absolute value of the variable x.

Absolute value of integers
The **absolute value** of an integer is its distance from **0** on the number line.



4 is 4 units away from 0, and -4 is also 4 units away from 0.

So, the absolute value of "4" is 4, and the absolute value of "-4" is also 4.

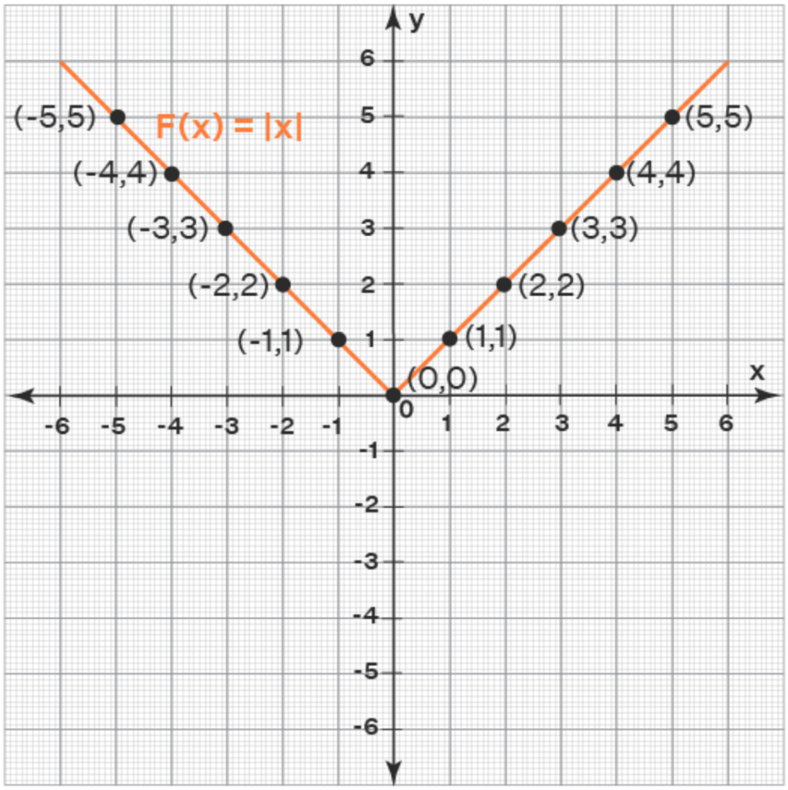
The absolute value function, written as $|x|$, is defined as follows:

- $|x| = x$ if x is greater than or equal to 0.
- $|x| = -x$ if x is less than 0.

Graphing of Absolute Value Function

Let us graph the simple function $f(x) =| x |$, which is defined as:
$f(x) = x,$ for $x \geq 0$
$f(x) = -x,$ for $x < 0$

x	**f(x) =| x |**
-4	4
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3
4	4



Sample Problems

Example Problem 1: Solve the equation $|2x - 5| = 11$.

Solution: To solve the equation $|2x - 5| = 11$, we consider two cases:
Case 1: $2x - 5 \geq 0$

$$2x - 5 = 11$$

$$2x = 11 + 5$$

$$2x = 16$$

$$x = \frac{16}{2} = 8$$

Case 2: $2x - 5 < 0$

$$-(2x - 5) = 11$$

$$-2x + 5 = 11$$

$$-2x = 11 - 5$$

$$-2x = 6$$

$$x = \frac{6}{-2} = -3$$

The solutions for equation $|2x - 5| = 11$ are $x = 8$ and $x = -3$.

Example Problem 2: Find the distance between -6 and 4 on the number line.

Solution: To find the distance between -6 and 4, we can use the absolute value.

$$|-6 - 4| = |-10| = 10$$

The distance between -6 and 4 on the number line is 10 units.

Example Problem 3: What is the ordered pair of real numbers (x, y) that satisfies equation $|x+y-7| + |4x-y+12| = 0$?

Solution: Since the absolute value of a number is always non-negative, we must have that $x+y-7=0$ and $4x-y+12=0$. Adding these equations together, we find $x=-1$. This means $y=8$, so the desired answer is $\boxed{(-1,8)}$.

1. Solve the following equations of absolute value: a). $|x| = 6$, then $x =$ ______________________ $|2x-3| = 9$, then $x =$ ______________________.

   b). $|2x-3| = 9$

   Case 1: $2x - 3 >= 0$ then

   $$2x-3=9,$$
   $$2x=12,$$
   $$x=6.$$

   Case 2: $2x-3<0$ then

   $$-(2x-3)=9,$$
   $$-2x+3=9,$$
   $$-2x=6.$$
   $$x=-3.$$

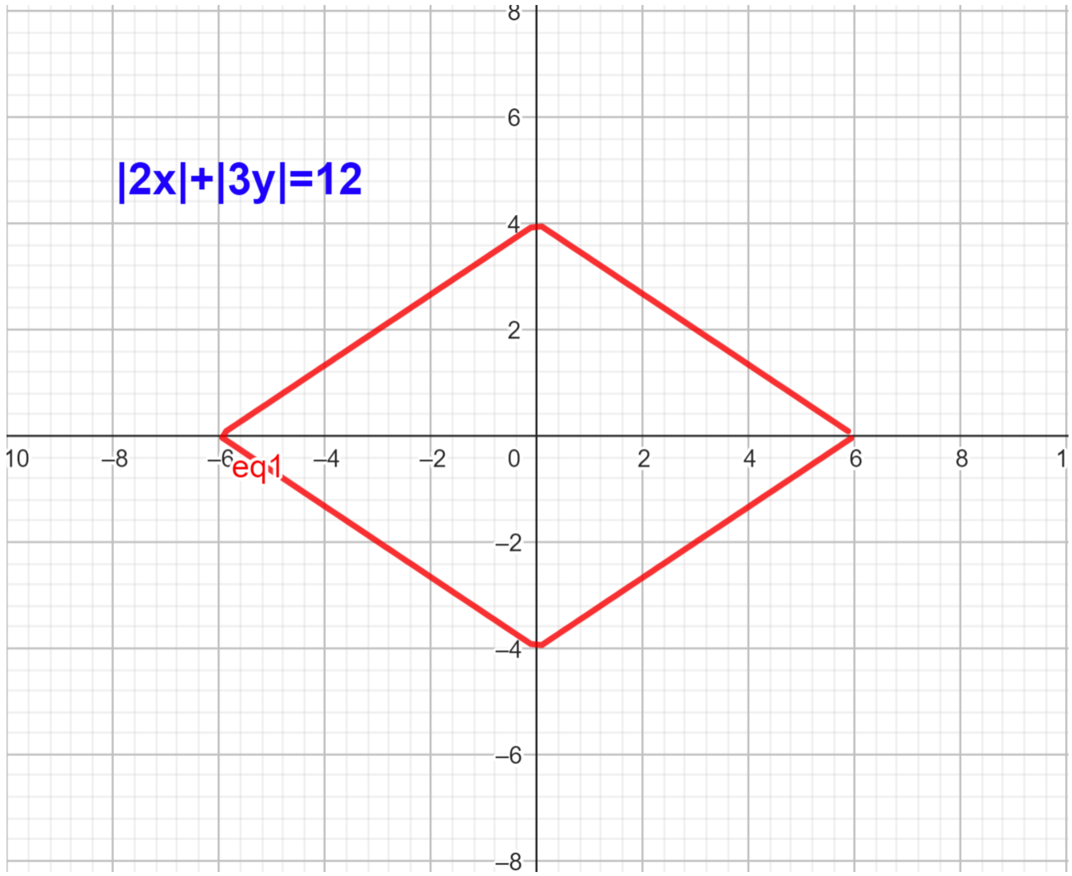
2. What is the area inside the graph of $|2x| + |3y| = 12$?
   Hint: Try graphing the equation by finding the x & y intercepts. What is the shape of the resulting graph?

   Solution: We will first try to find the bounds of the shape by finding the x&y intercepts.

   Case x-intercept $(y=0)$**:**
   We have $|2x| = 12 \longrightarrow x=6$ or $x=-6$

   Case y-intercept $(x=0)$**:**
   We have $|3y| = 12 \longrightarrow y=4$ or $x=-4$

   Now, we graph the above x and y values as shown in the below diagram.



As we can see, this is basically a quadrilateral composed of 4 right triangles with a base of 6 units and a height of 4 units.

Area of each triangle : $\frac{6 \cdot 4}{2} = 12$ sq. units

Area of quadrilateral: $\boxed{4 \cdot 12 = 48}$. sq. units

3. What is the area inside the graph of $|2x - 4| + |3y - 6| = 12$?
Solution: Instead of finding the x & y intercepts, we consider the following two cases:

Case $3y - 6 = 0$:

$$3y - 6 = 0 \Longrightarrow y = 2$$

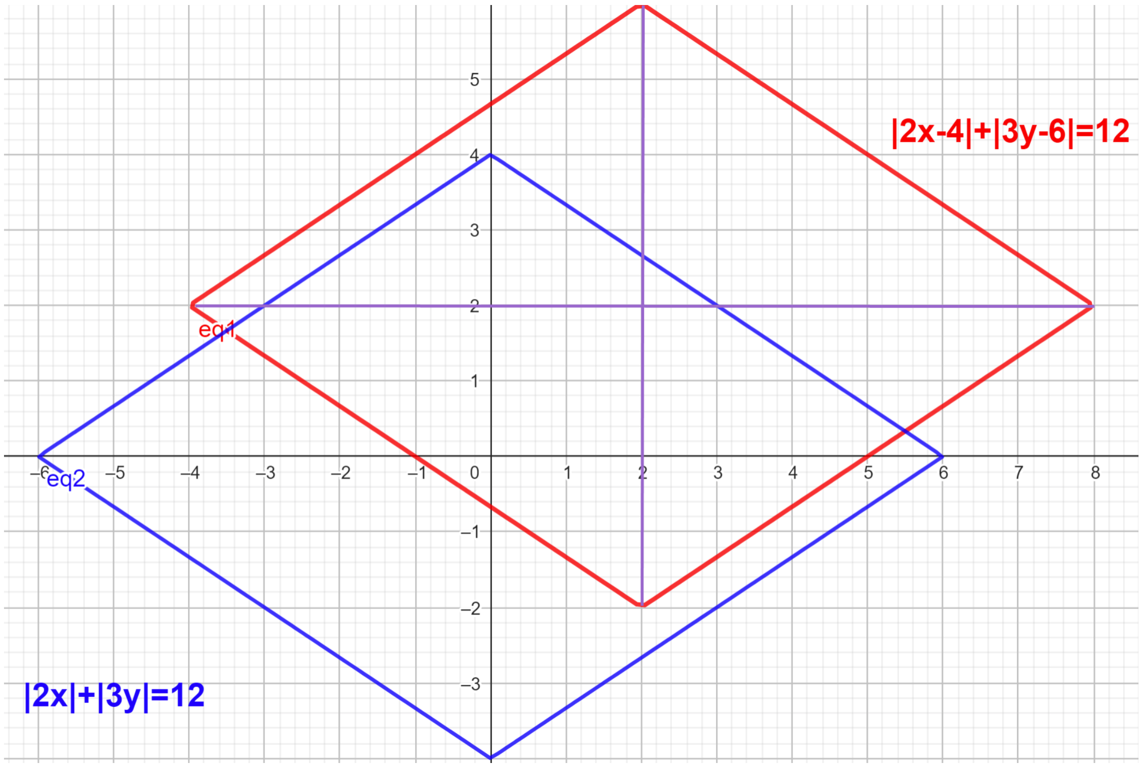
,
We have $2x - 4 = 12 \Longrightarrow x = 8$
or $-(2x - 4) = 12 \Longrightarrow x = -4$

Case $2x - 4 = 0$:

$$2x - 4 = 0 \Longrightarrow x = 2,$$

We have $3y - 6 = 12 \Longrightarrow y = 6$
or $-(3y - 6) = 12 \Longrightarrow y = -2$

Now, we graph the above x and y values as shown in the below diagram.



As we can see in the diagram in Red, this is basically a quadrilateral composed of 4 right triangles with a base of 6 units and a height of 4 units similar to the diagram from the previous screen (shown here in Blue) except that the diagram is shifted by 2 units up and 2 units to the right.

Area of each triangle : $\frac{6 \cdot 4}{2} = 12$ sq. units

Area of quadrilateral: $\boxed{4 \cdot 12 = 48}$. sq. units

2 Homework

Solve for x in the following equations:

1. $|5x + 2| = 18 \longrightarrow x =$
2. $-|x| = x - 10 \longrightarrow x =$
3. $7 + 5 \cdot |3x| = 37 \longrightarrow x =$
4. $3|x + 5| = 6 \longrightarrow x =$
5. $3|x + 2| - 5 = 4 \longrightarrow x =$